Assignment 6

Network Flow

Task 1 Flow Networks

a) **List all the s-t cuts in Flow Network Alpha and their capacities. Do this by describing the two sets of nodes the cut creates (For instance ($\{s\}$, $\{x_1, x_2, t\}$), *and* by drawing the cut graph(s).**

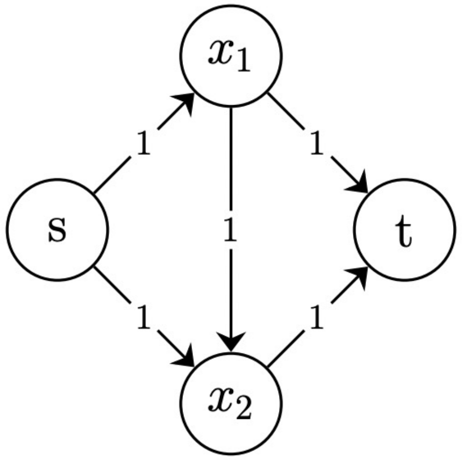

Figure 1: Flow Network Alpha

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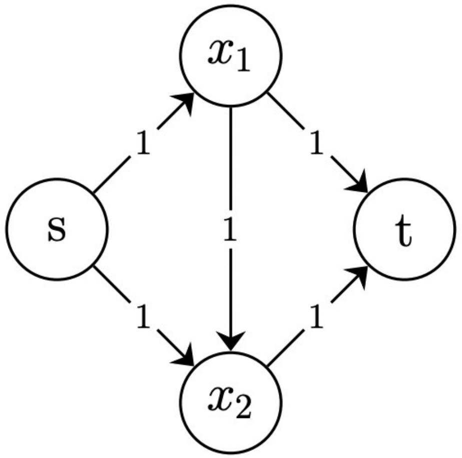

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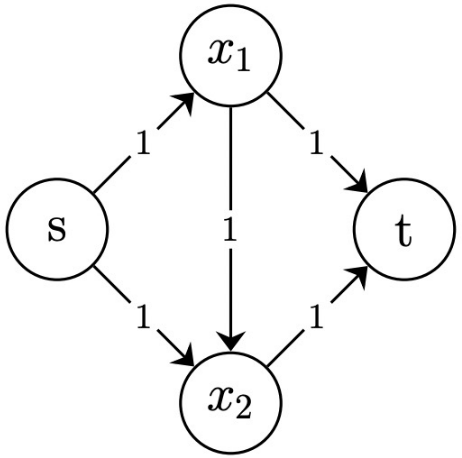

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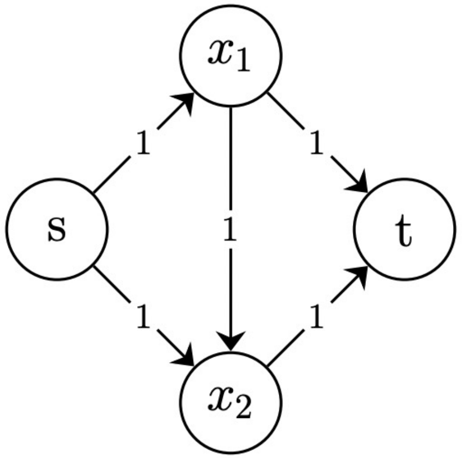
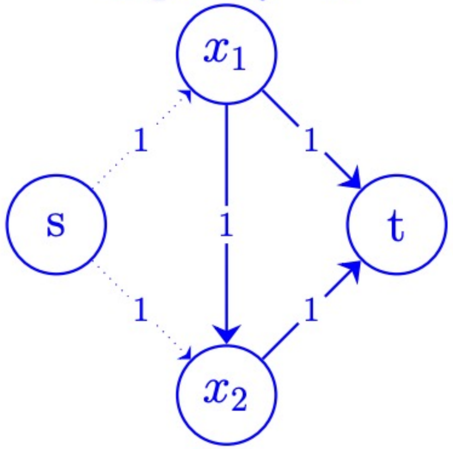


Figure 1: Flow Network Alpha

($\{s\}$, $\{x_1, x_2, t\}$)
capacity=2



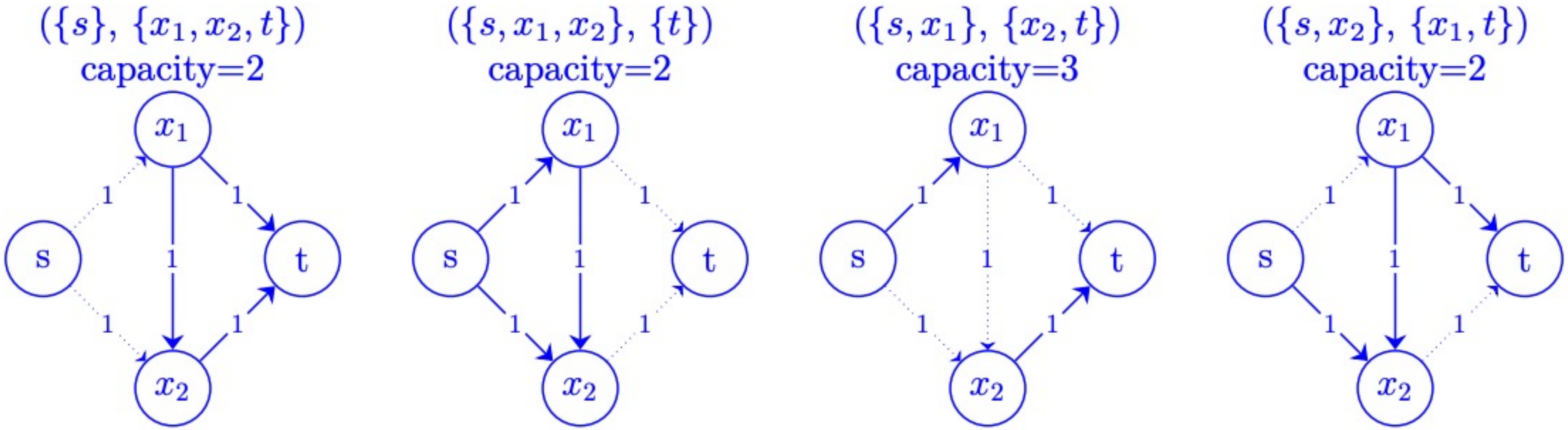
Set (A,B) such that there is no path from s to t.

Set A: Nodes reachable from s after the cut
Set B: Nodes **not** reachable from s after the cut

How to:

- **Delete edges until you have such a set**
- **Capacity: Sum of the deleted edges (with weights)**

There are four such cuts



$(\{s\},\ \{x_1, x_2, t\})$
capacity=2
$(\{s, x_1, x_2\},\ \{t\})$
capacity=2
$(\{s, x_1\},\ \{x_2, t\})$
capacity=3
$(\{s, x_2\},\ \{x_1, t\})$
capacity=2
x_1
s
t
x_2
1

b) What is the minimum s-t cut in Flow Network Bravo? What is its capacity?

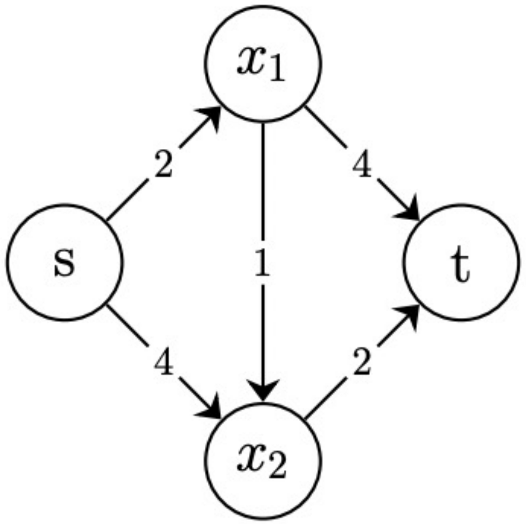


Figure 2: Flow Network Bravo

b) What is the minimum s-t cut in Flow Network Bravo? What is its capacity?

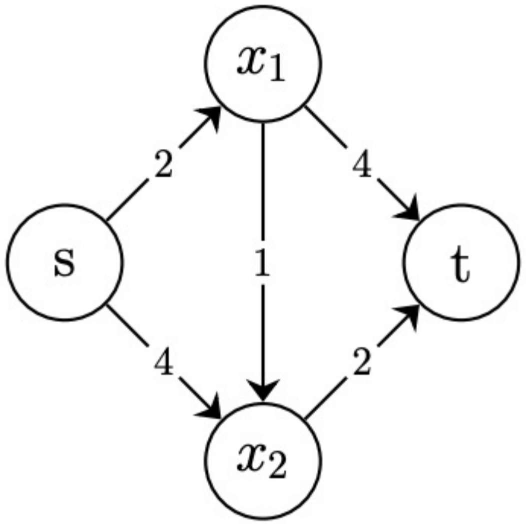
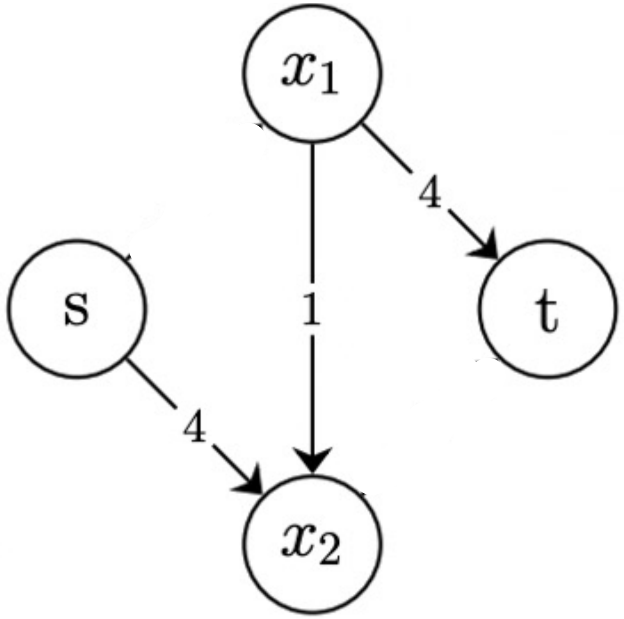


Figure 2: Flow Network Bravo



Solution:
The minimum capacity of an s-t cut is **4** with the cut $(\{s, x_2\}, \{x_1, t\})$.

c) What is the value of the computed flow in Flow Network Charlie? Is this a maximum (s, t) flow in this graph? Explain your answers.

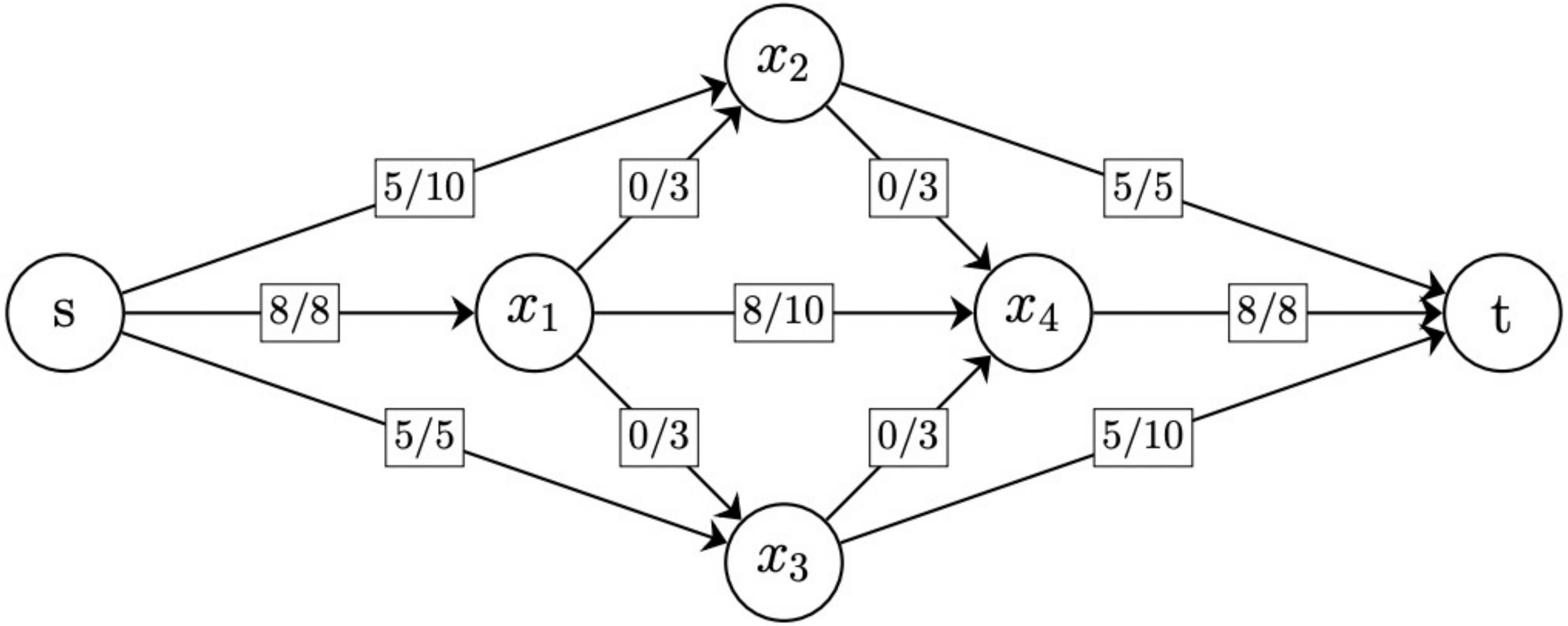


Figure 3: Flow Network Charlie. The flow and capacity of each edge appears as a boxed label on the edge in the form [$flow/capacity$].

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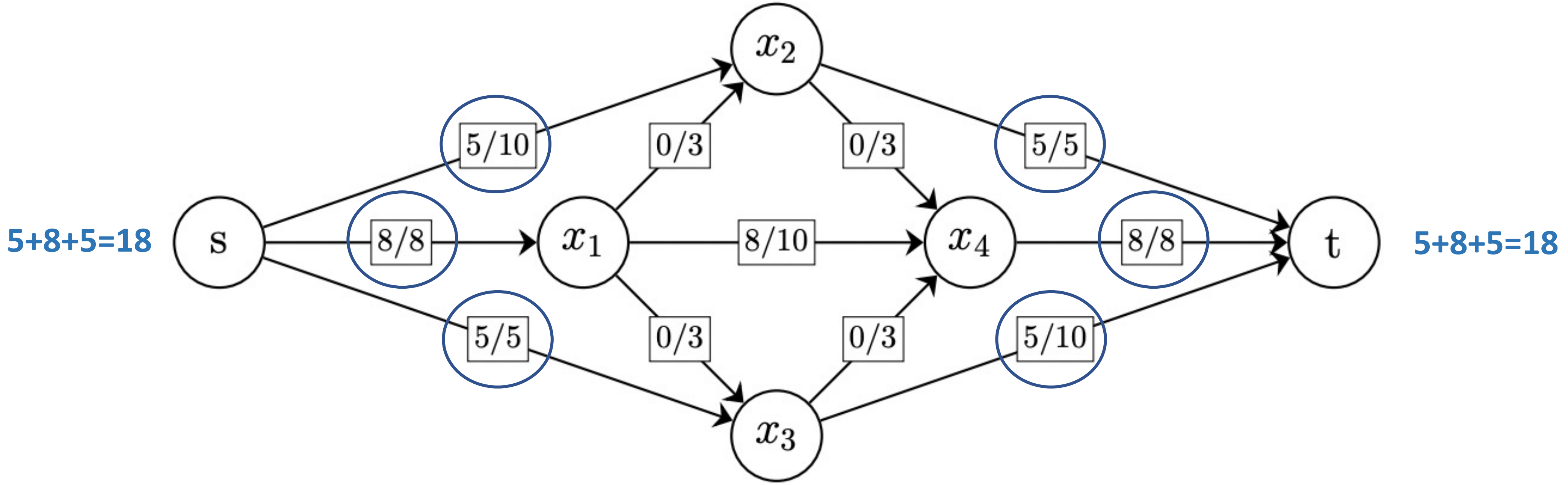

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Solution:

The calculated flow has a value of 18. (we can simply check the amount of flow out of the source node, or into the sink node $5 + 5 + 8 = 18$). It is **not** a maximum (s, t) flow for this graph.

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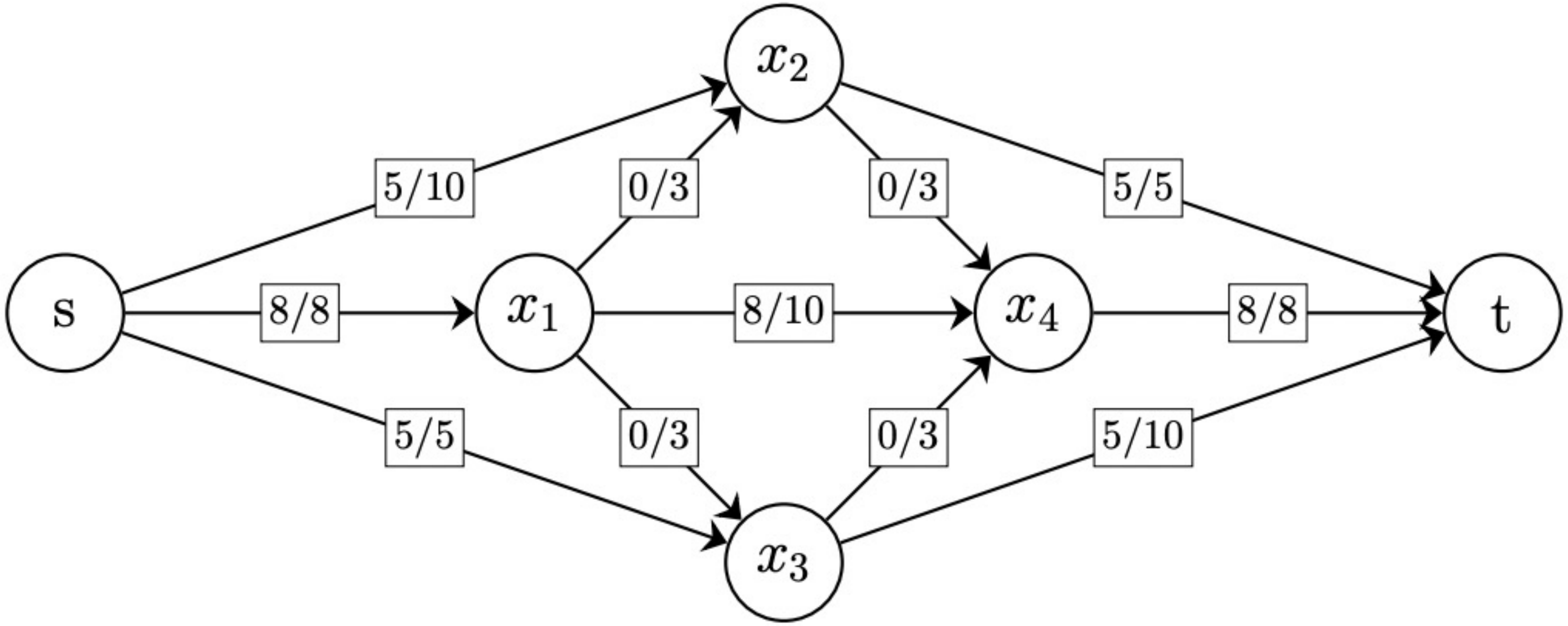

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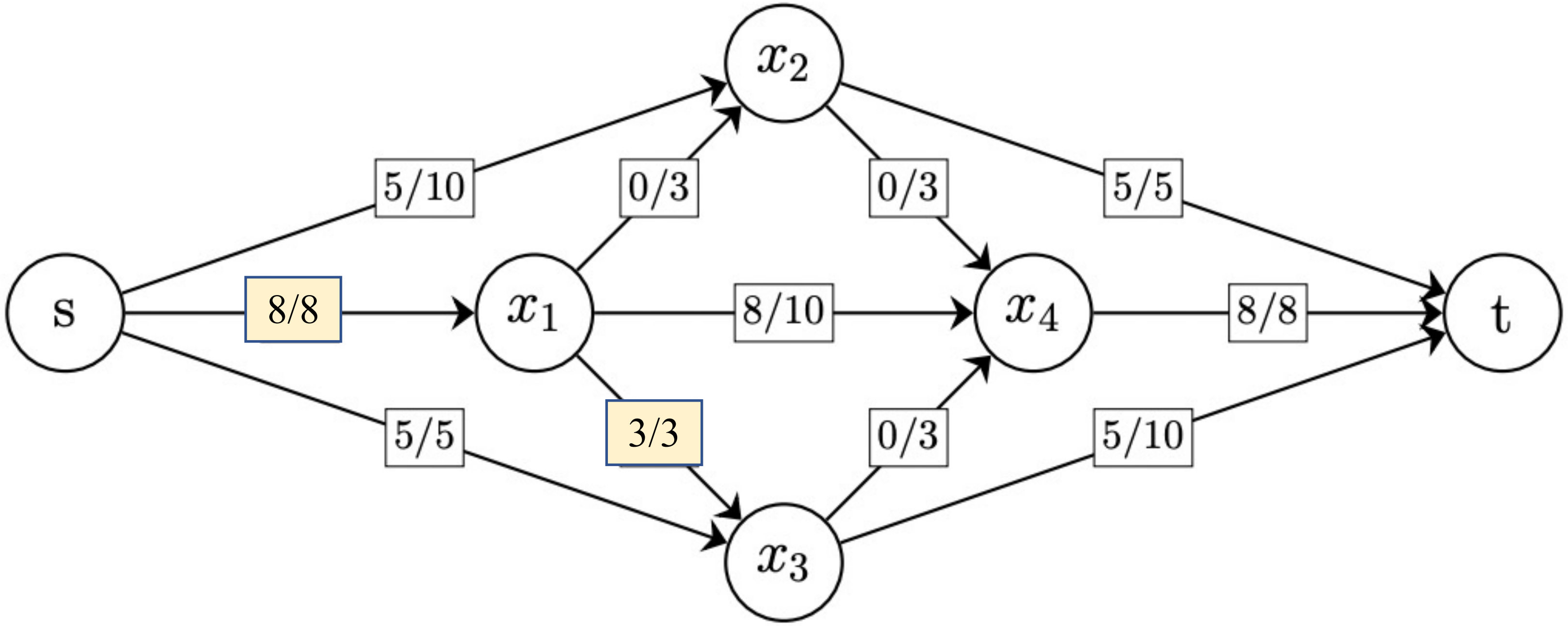

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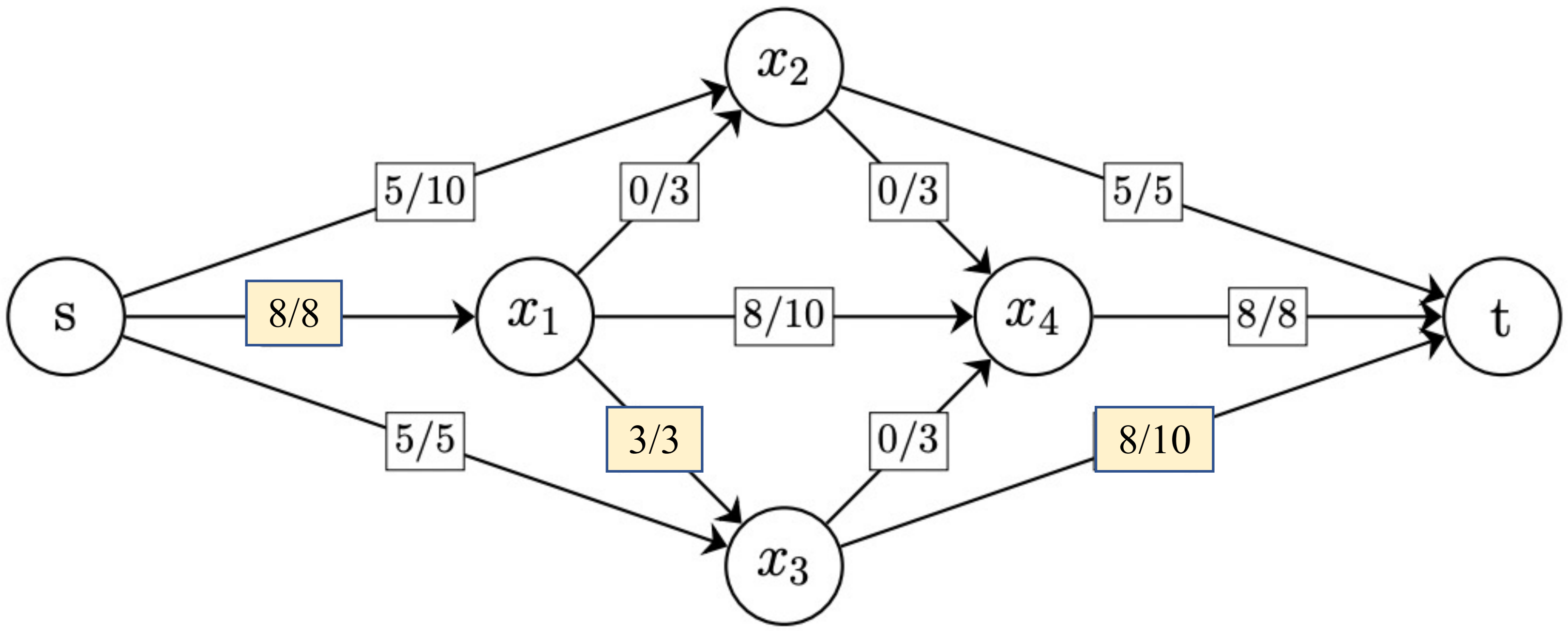

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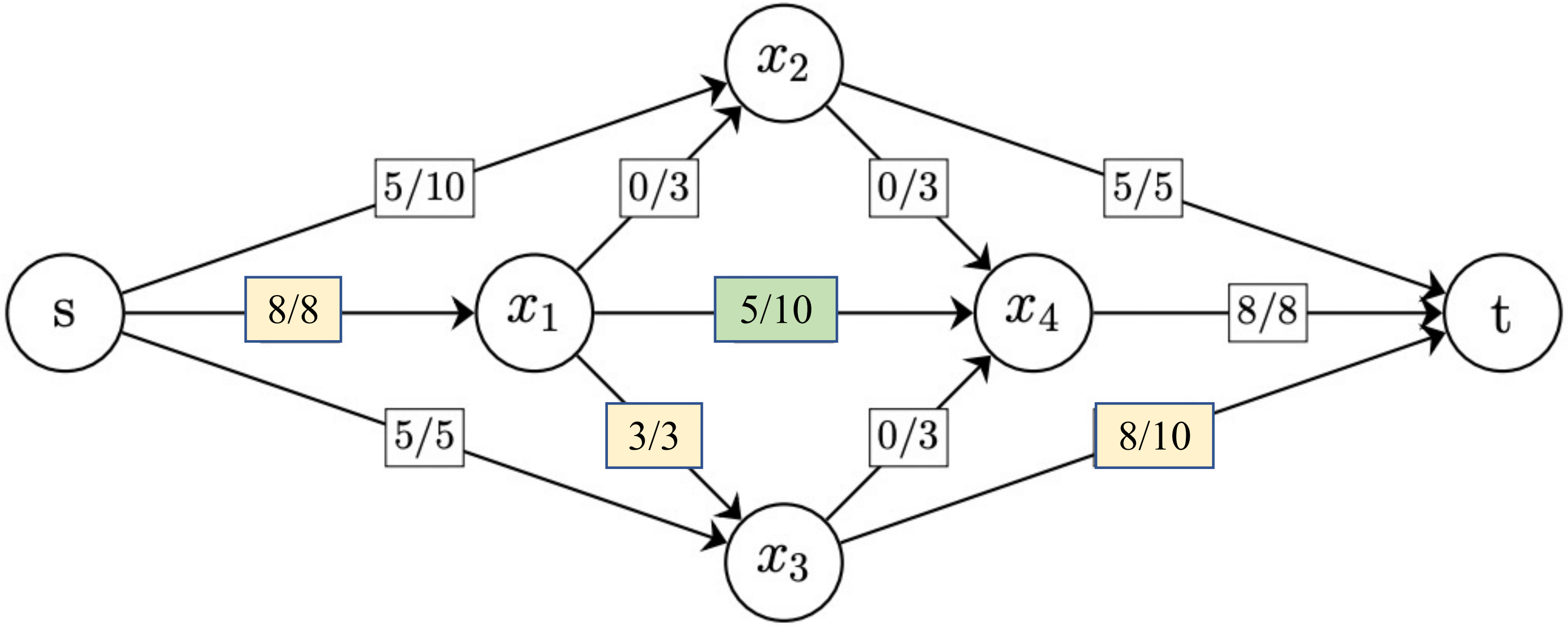

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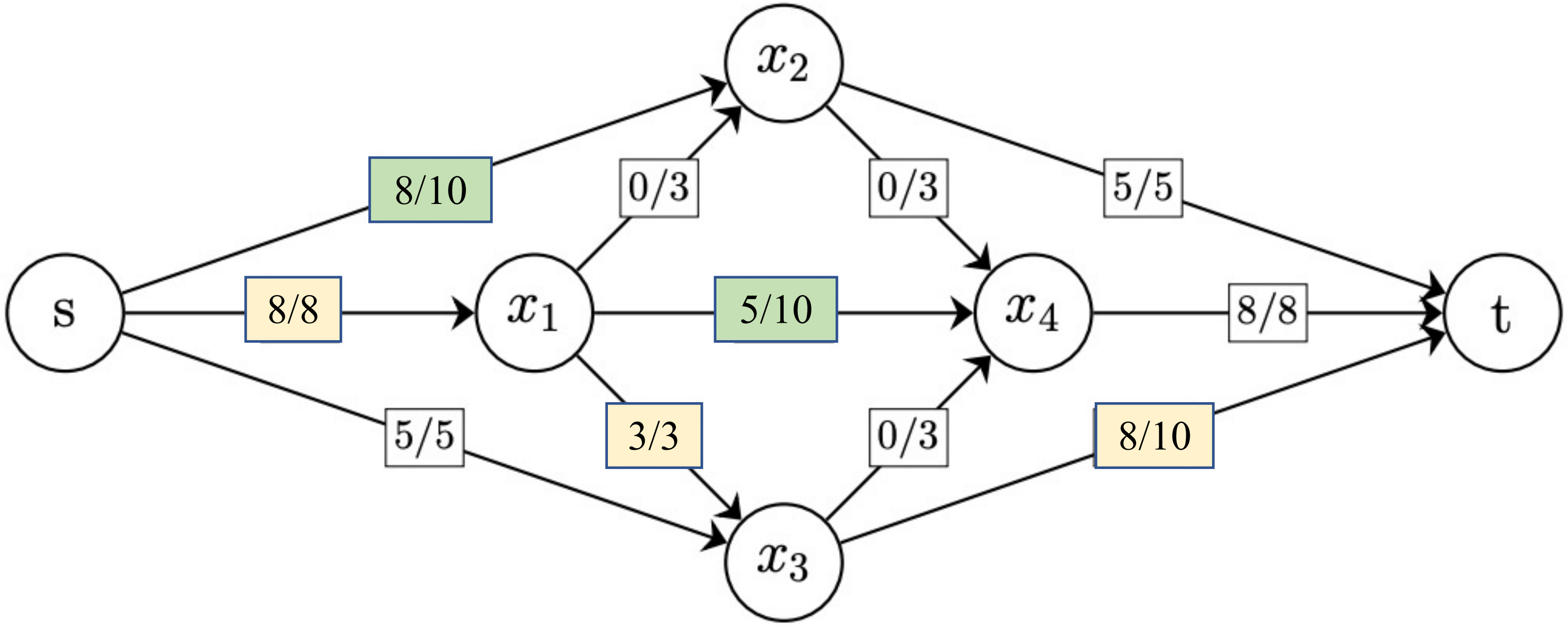

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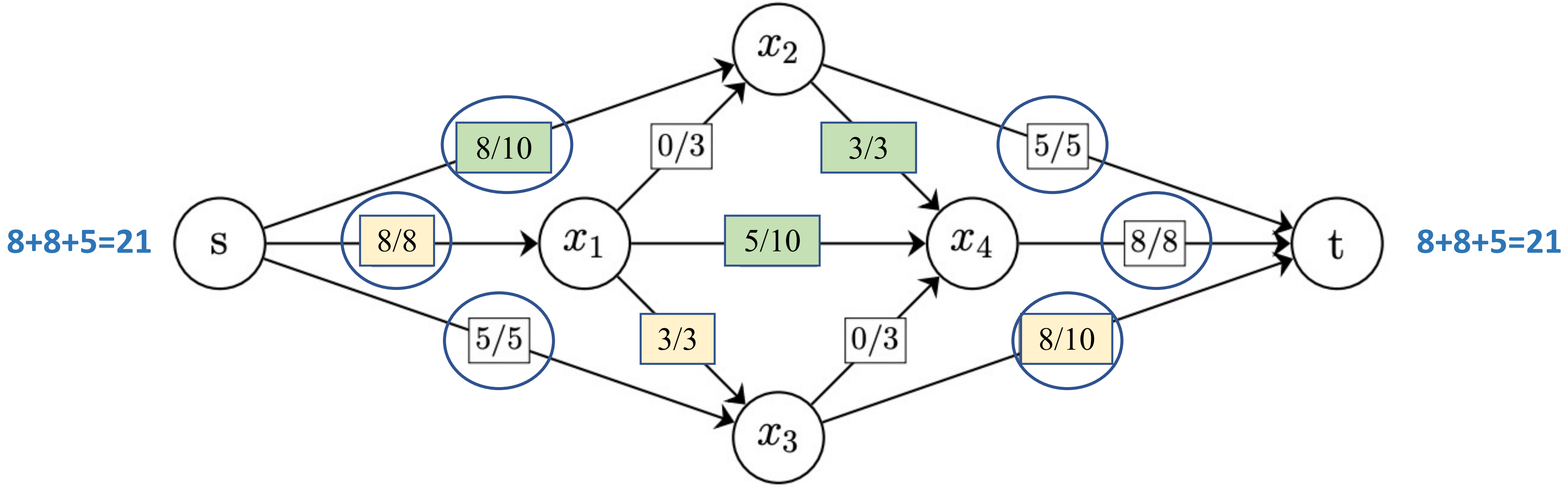

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d) Find a minimum s-t cut in Flow Network Delta. What is its capacity?

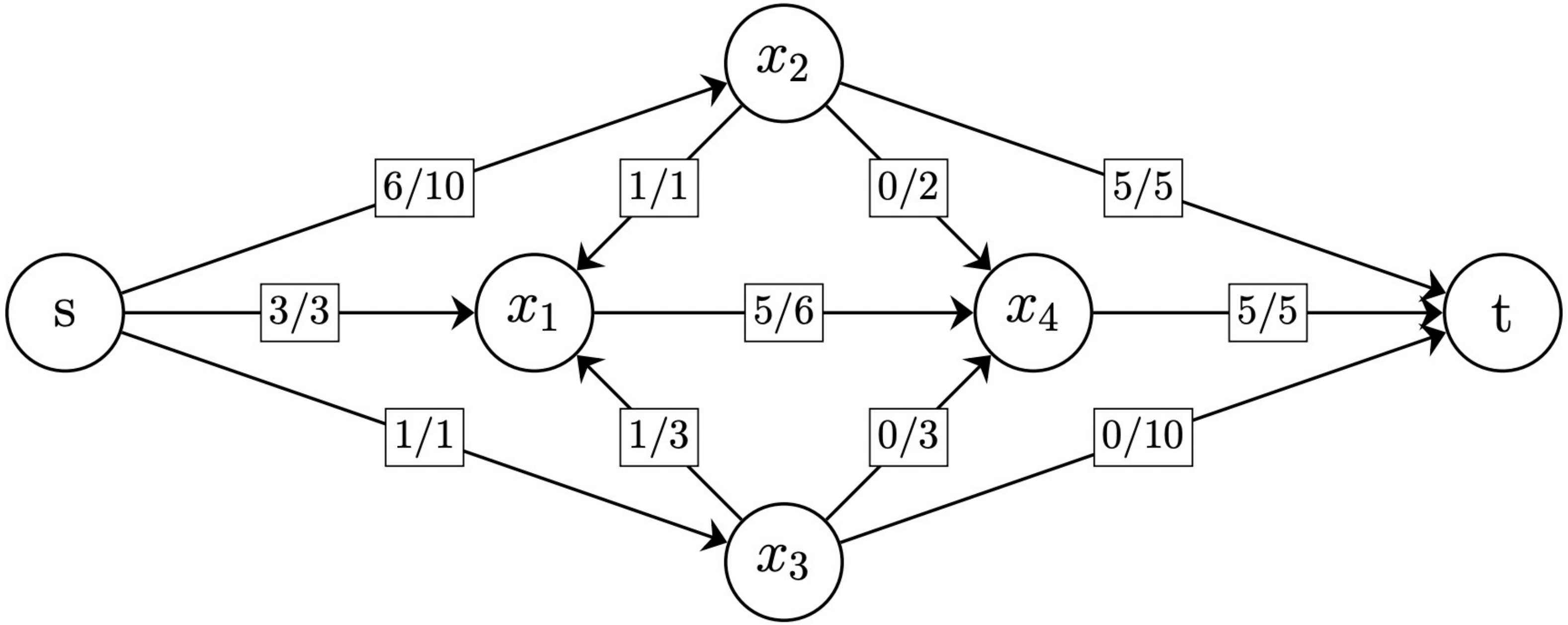

Figure 4: Flow Network Delta

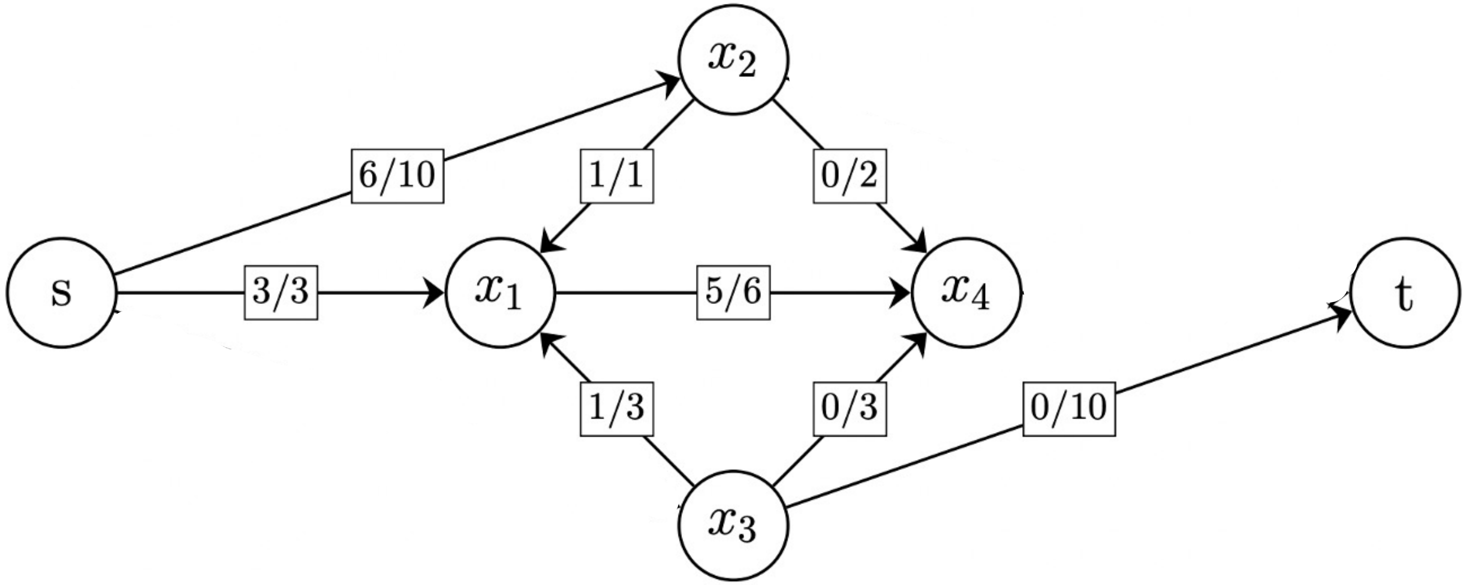

Figure 4: Flow Network Delta

we cut the following edges:

$$
\begin{aligned}
&(s, x_3) \quad (capacity = 1) \\
&(x_2, t) \quad (capacity = 5) \\
&(x_4, t) \quad (capacity = 5)
\end{aligned}
$$

$(\{s, x_1, x_2, x_4\}, \{x_3, t\})$ is a minimum cut with capacity **11**.

e) Decide whether you think the following statements are true or false. If they are true, give a short explanation. If they are false, give a counterexample.

Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e.

i. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).

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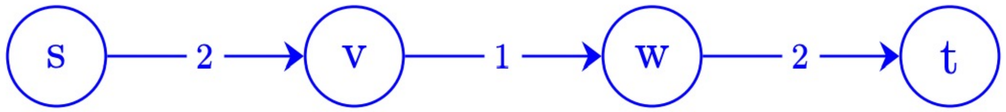

The maximum flow is 1, but the capacity out of s is 2.

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Solution:
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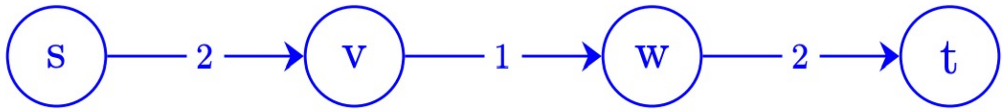

The maximum flow is 1, but the capacity out of s is 2.

ii. Let $(A,\ B)$, where A and B are the two sets of nodes that the minimum cut splits G into, be a mimimum s-t cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then $(A,\ B)$ is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

ii. Let (A, B), where A and B are the two sets of nodes that the minimum cut splits G into, be a mimimum s-t cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

Lets first simplify the statement. It says that if a graph has a minimum cut between A and B, this will still be the minimum cut if we increase all edge capacities with 1.

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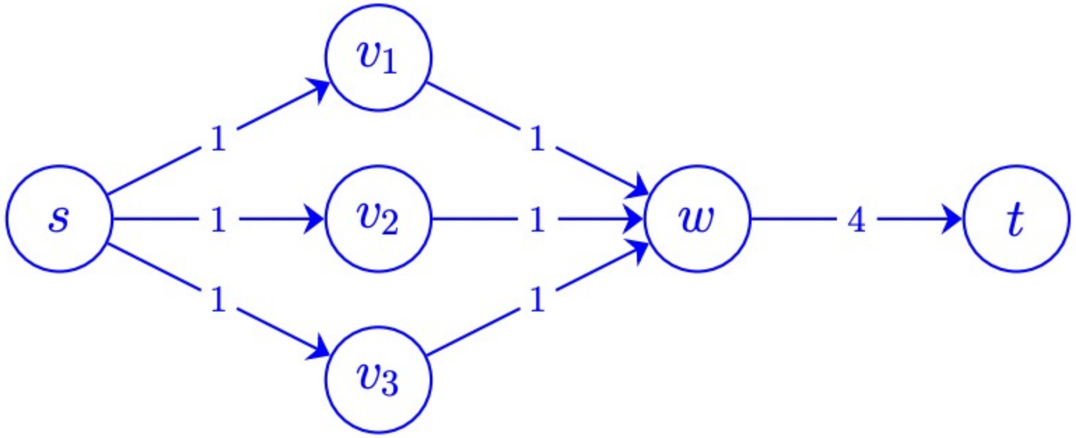

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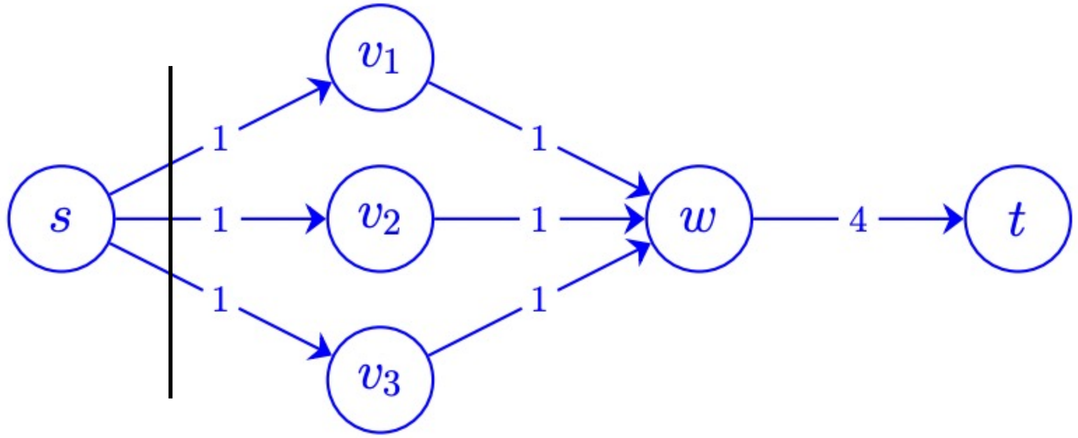

This graph has minimum cut of 3 between s and w.

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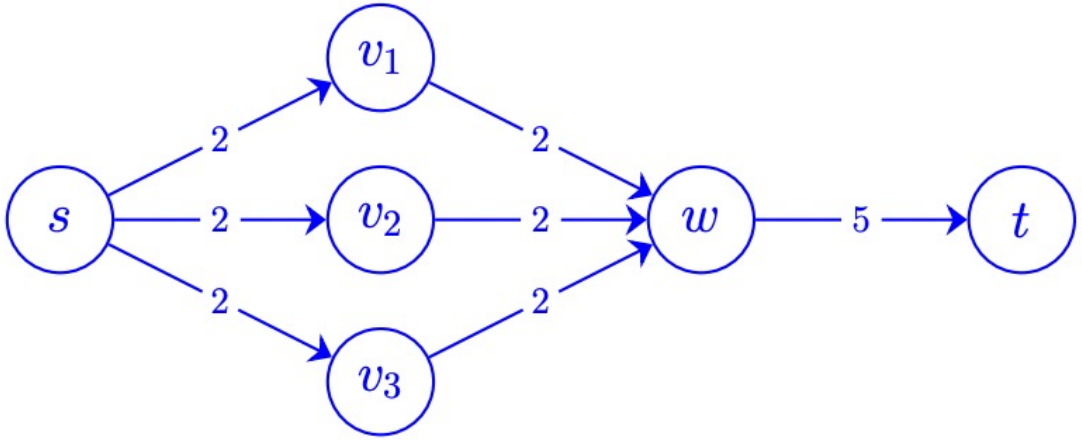
Now lets add 1 to each edges capacity:



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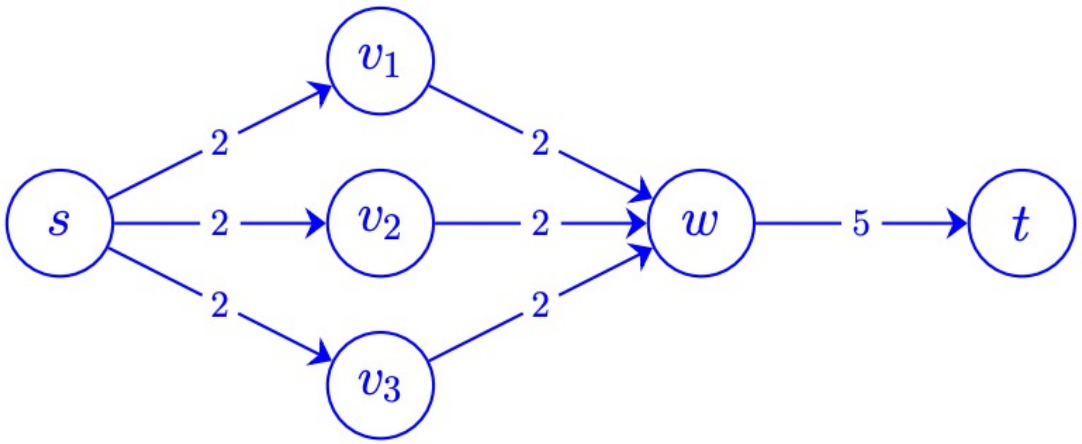

The graph now has a minimum cut of 5 between w and t ($A = \{s, v, w\}$, $B = \{t\}$). This is not the same minimum cut as before, thus disproving the statement.

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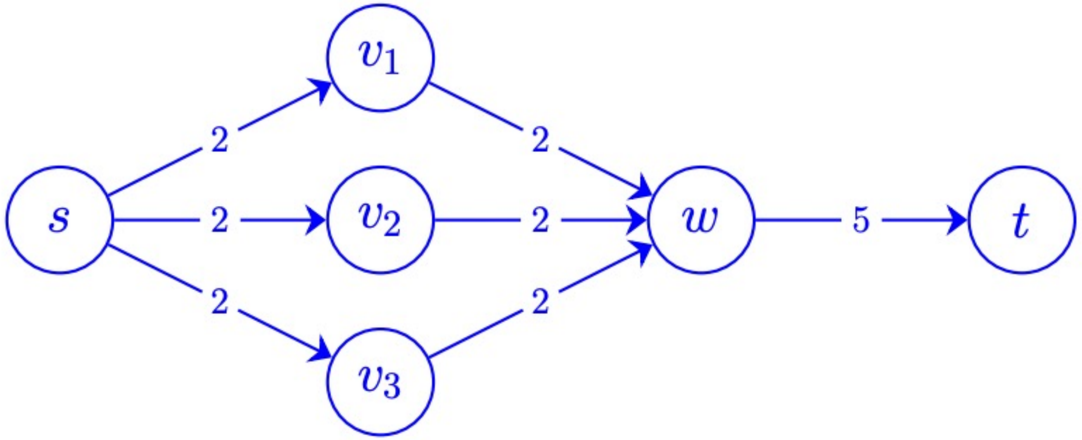

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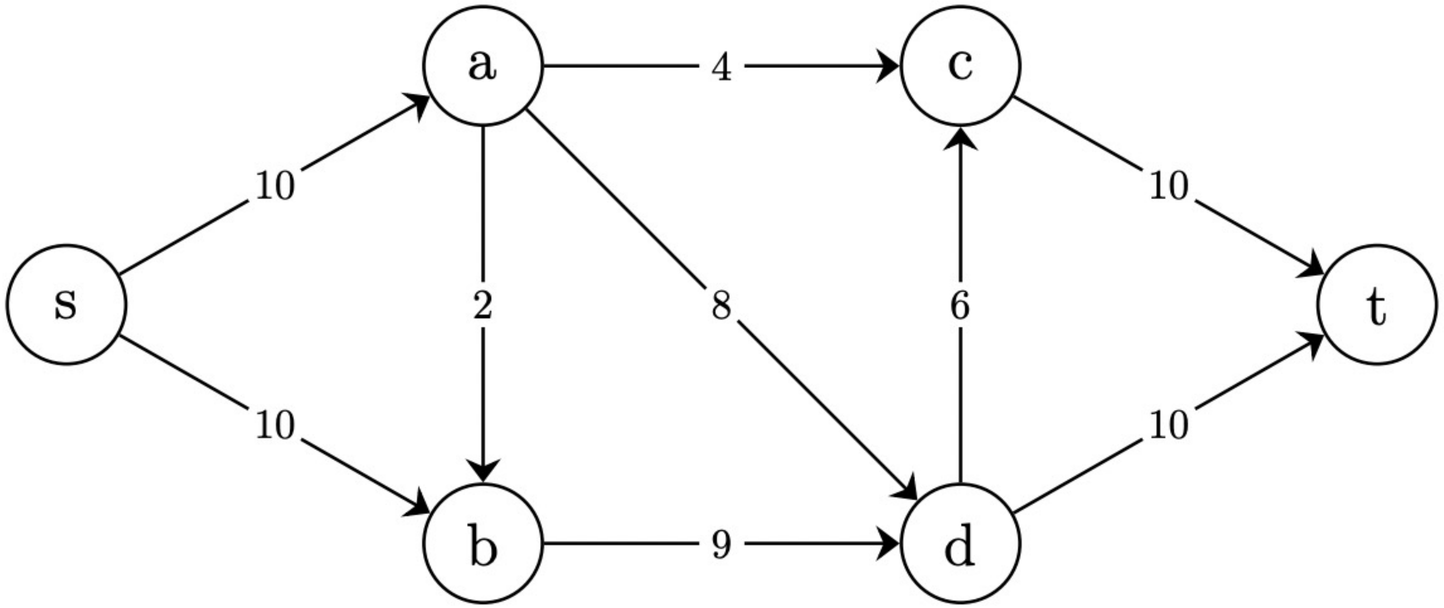
Solution:
False.

Task 2 Ford-Fulkerson

Given the following graph with capacities:



a) Apply the Ford-Fulkerson algorithm manually to find the maximum flow in this network. Show all augmenting paths. What is the Maximum Flow?

Task 3 Blood Donors

Statistically, the arrival of spring typically results in increased accidents and increased need for emergency medical treatment, which often requires blood transfusions. Consider the problem faced by a hospital that is trying to evaluate whether its blood supply is sufficient.

The basic rule for blood donation is the following. A person's own blood supply has certain *antigens* present (we can think of antigens as a kind of molecular signature); and a person cannot receive blood with a particular antigen if their own blood does not have this antigen present. Concretely, this principle underpins the division of blood into four types: A, B, AB, and O. Blood of type A has the A antigen, blood of type B has the B antigen, blood of type AB has both, and blood of type O has neither. Thus, patients with type A can receive only blood types A or O in a transfusion, patients with type B can receive only B or O, patients with type O can receive only O, and patients with type AB can receive any of the four types.

a)

Let s_O, s_A, s_B, and s_{AB} denote the supply in whole units of the different blood types on hand. Assume that the hospital knows the projected demand for each blood type d_O, d_A, d_B, and d_{AB} for the coming week. Explain how a flow algorithm can be used to evaluate if the blood on hand would suffice for the projected demand.

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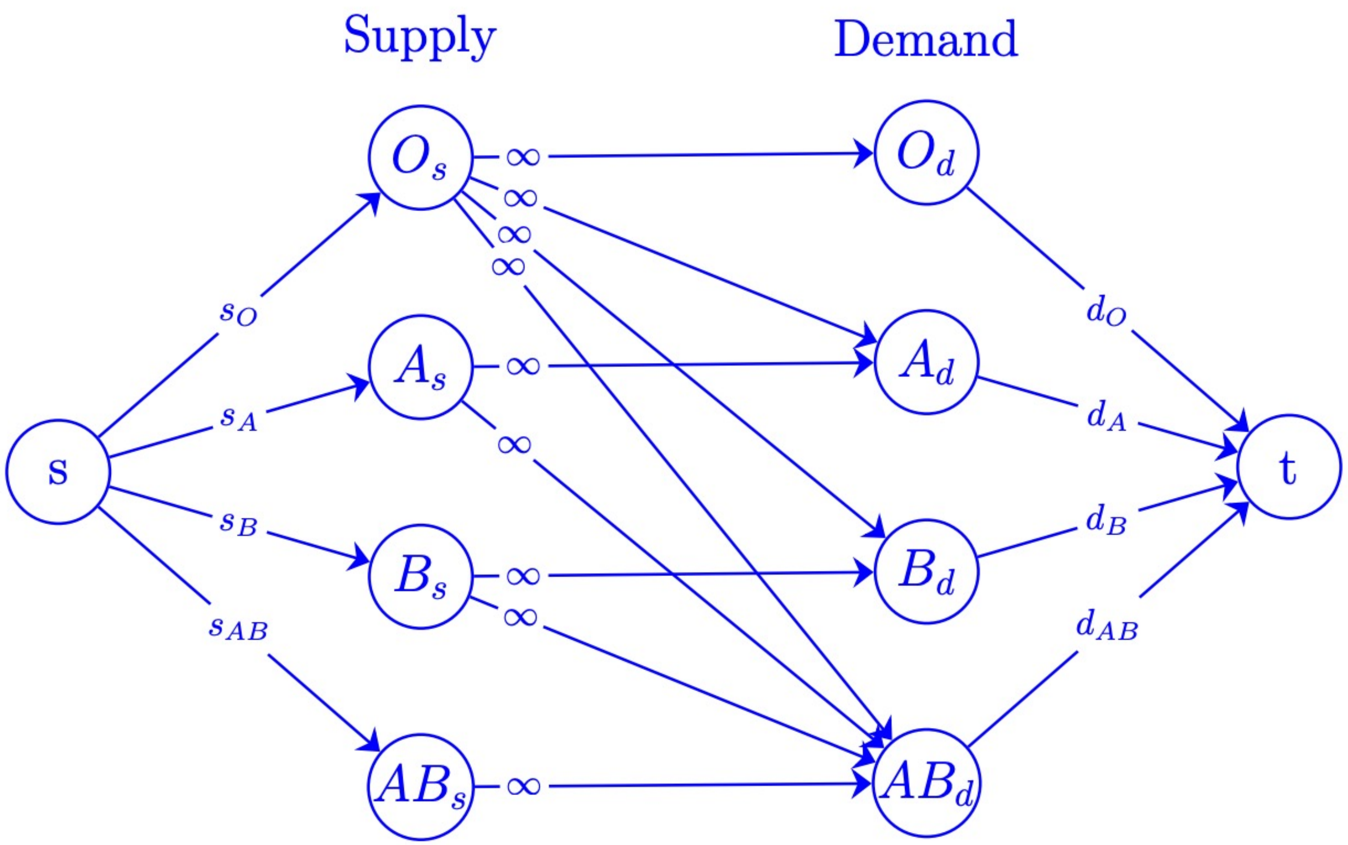
- The hospital has an overall supply and demand for blood
- You have donors and recievers (green)
- You have an constraint between these groups (yellow)

How can each of theese bullets be translated to a network flow?



We can now simply perform any maximum flow algorithm.

We know the demand is met if all the edges going directly into the sink (t) are satureated.

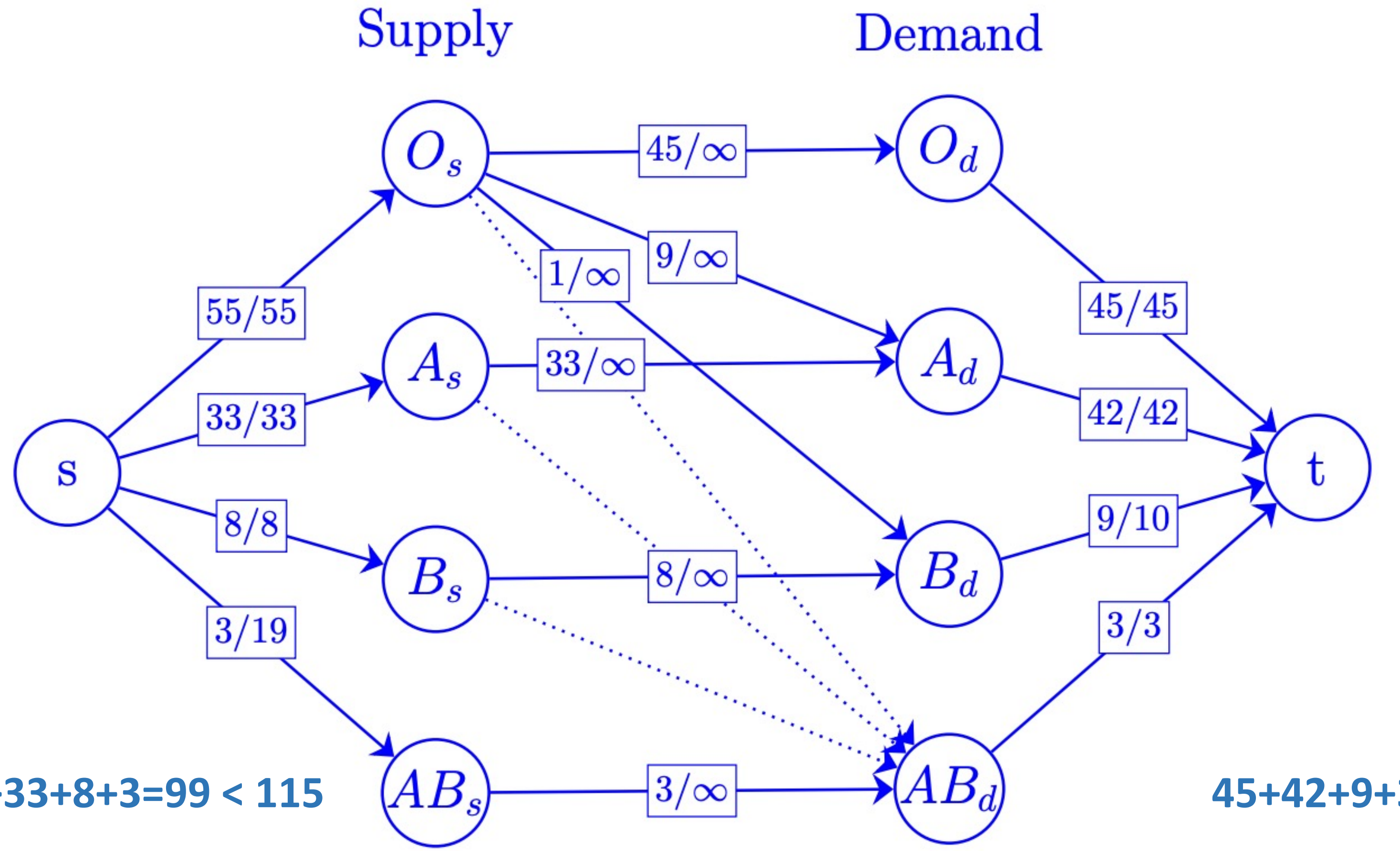
b)

Over the next week, they expect to need at most 100 units of blood. The typical distribution of blood types in U.S. patients is roughly 45 percent type O, 42 percent type A, 10 percent type B, and 3 percent type AB. The hospital wants to know if the blood supply it has on hand would be enough if 100 patients arrive with the expected type distribution. There is a total of 115 units of blood on hand. The table below gives these demands, and the supply on hand.

Blood Type	Supply	Demand
O	55	45
A	33	42
B	8	10
AB	19	3

i. Is the 115 units of blood on hand enough to satisfy the 100 units of demand? Find an allocation that satisfies the maximum possible number of patients.

We simply perform the algorithm described in the previous task. This gives us the following maximum flow (allocation that satisfies the maximum number of patients). Paths with no flow are shown with dotted lines.



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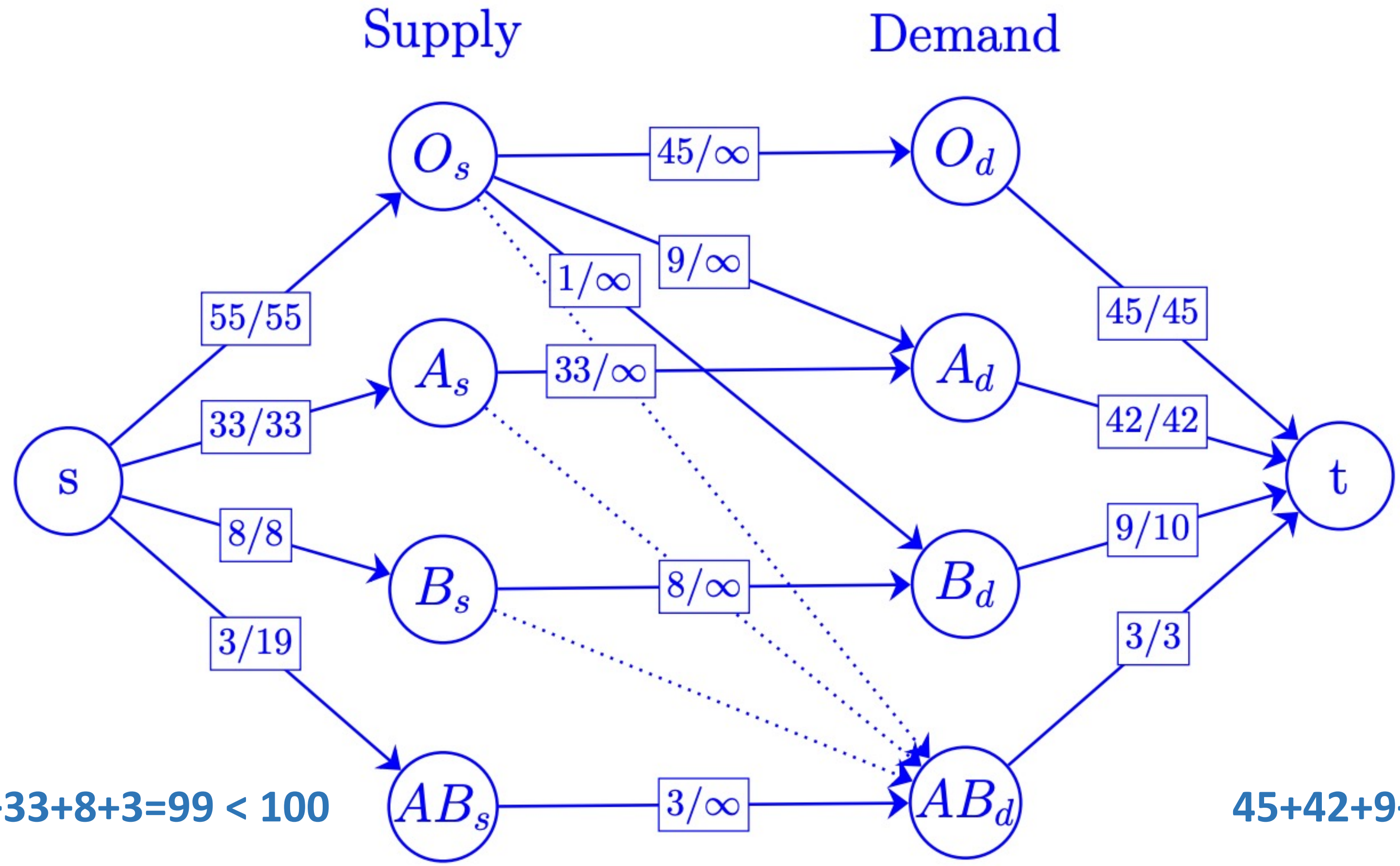
55+33+8+3=99 < 115

45+42+9+3=99 < 115



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O	55	45
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55+33+8+3=99 < 100

45+42+9+3=99 < 100

Solution:
No, there is not enough supply of blood.

ii. Use an argument based on a minimum-capacity cut to show why not all patients can receive blood. Also, provide an explanation for this fact that would be understandable to the clinic administrators, who have not taken a course on algorithms. (So, for example, this explanation should not involve the words *flow*, *cut*, or *graph*).

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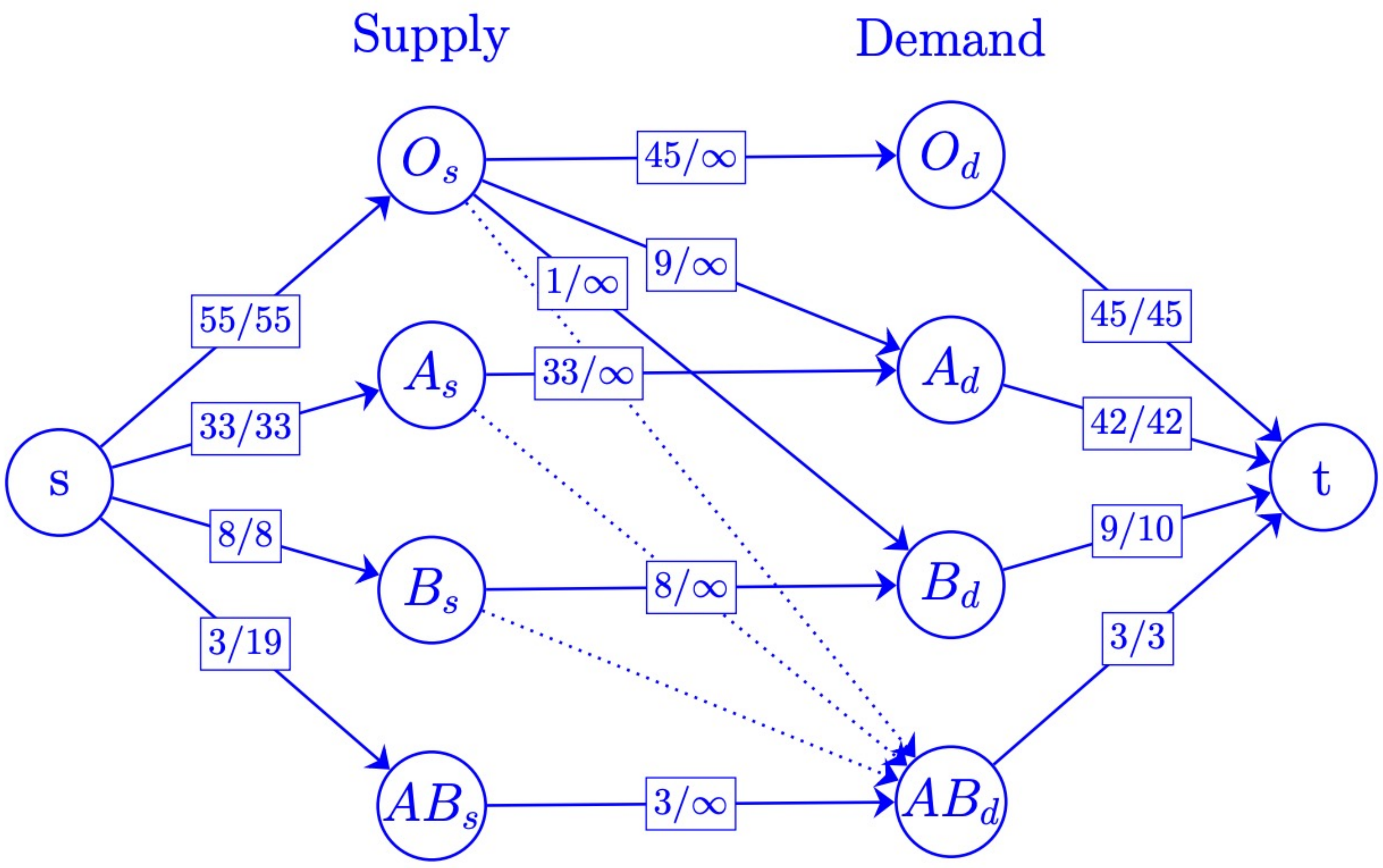
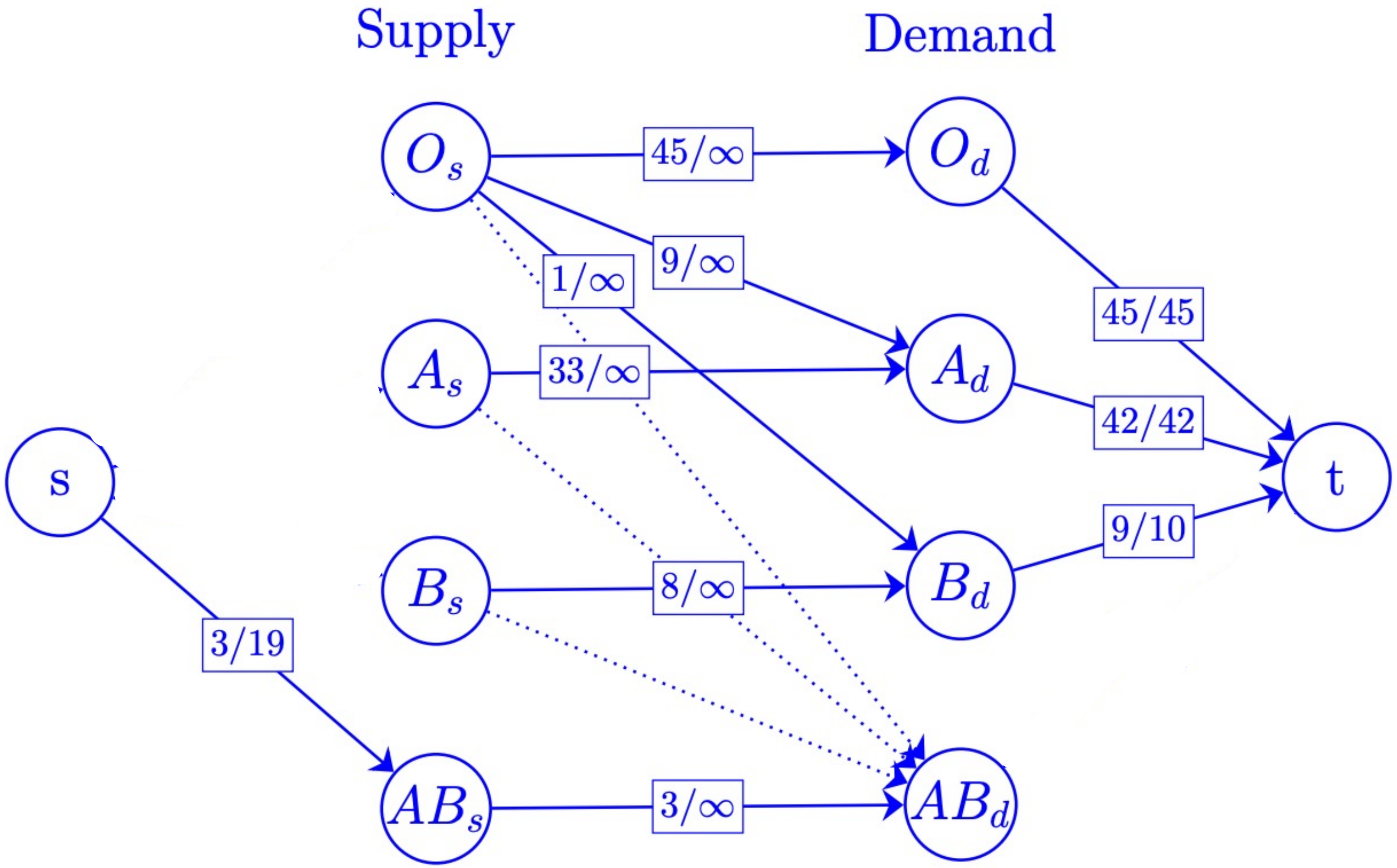


Solution:
The minimum cut on this flow network will be cutting this set of edges: $\{(s, O_s), (s, A_s), (s, B_s), (AB_d, t)\}$ which has a capacities of $55 + 33 + 8 + 3 = 99$ which is lower than the required capacity of 100.



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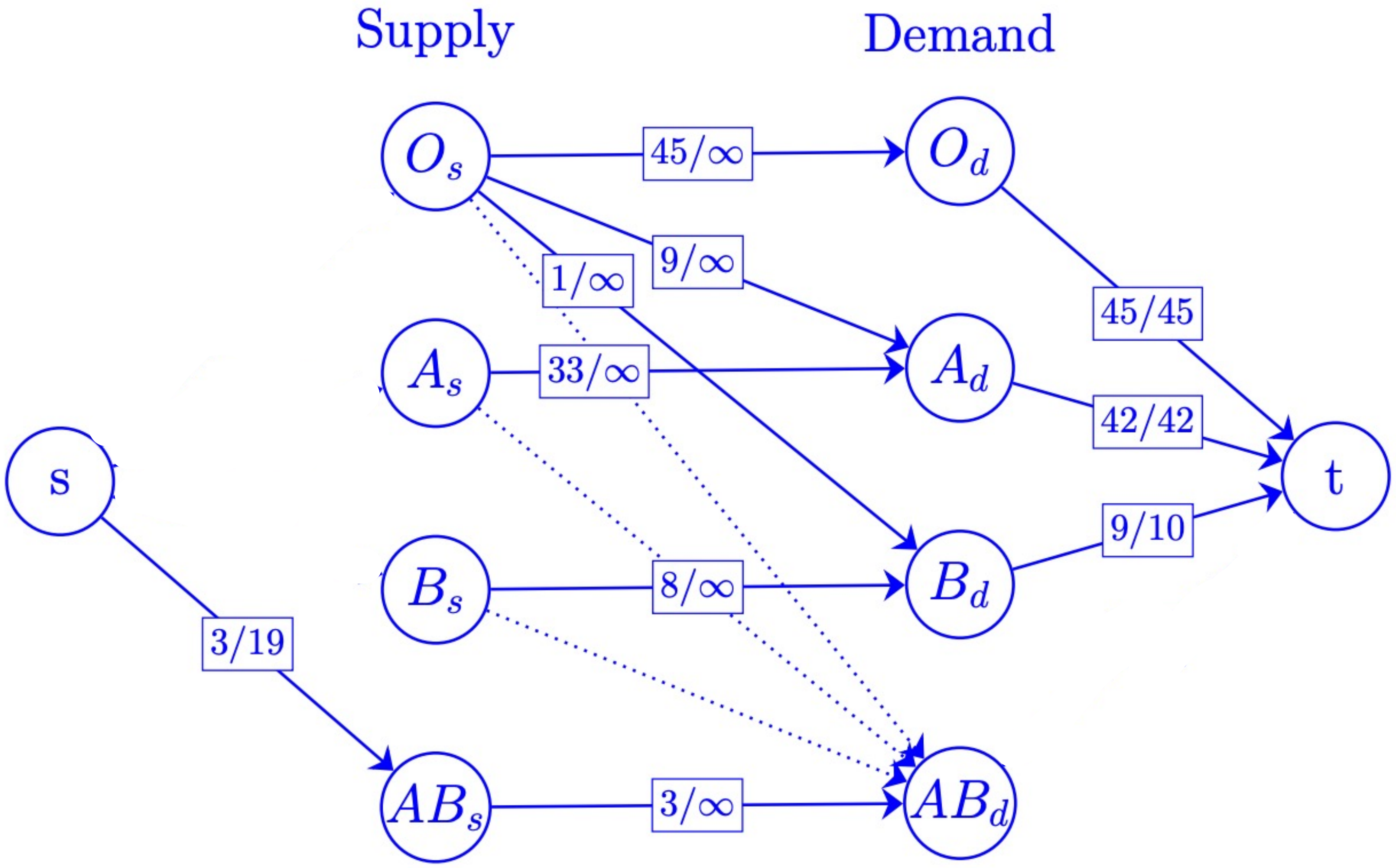

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To explain it to the clinic administrators we could say that there is 97 people with demand for blood type A, B or O, but there is only 96 people that can donate these blood types. (a new donor of either blood type can fix the shortage).