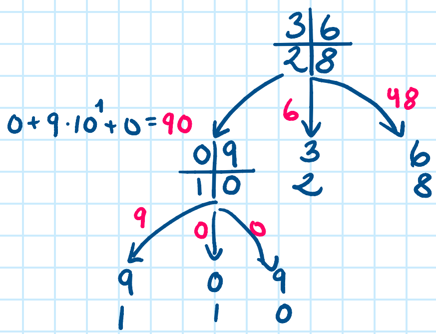
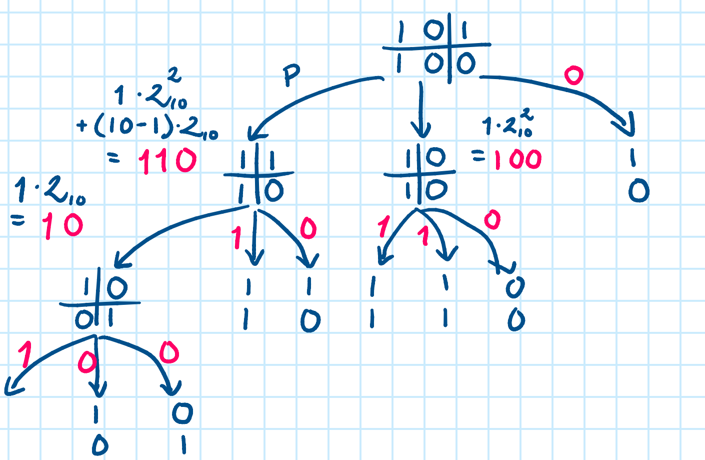
Lecture 28/09

Wednesday, 28 September 2022 00:47

1) $6 \cdot 10^2 + (90 - 6 - 48) \cdot 10^1 + 48 = \underline{1008}$

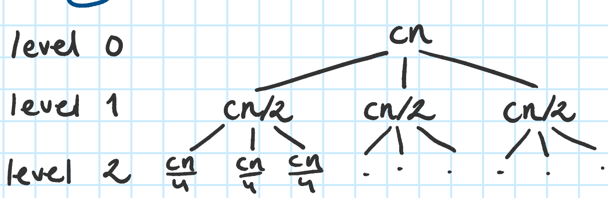


2) $100 \cdot 2_{10}^2 + (110 - 100) \cdot 2_{10}$
$= 10000 + 100 = \underline{10100}$



3) $T(n) = 3T(n/2) + cn$

1



2

cn

3cn/2

9cn/4

$\log_2 n$

3 $T(n) = cn \sum_{j=0}^{\log n} \left(\frac{3}{2}\right)^j$

$= cn \left(\frac{1 - (3/2)^{\log n}}{1 - (3/2)} \right)$

$= 2cn \left((3/2)^{\log n} - 1 \right)$

$= 2cn \cdot n^{\log(3/2)} - 2cn$

$= 2cn \cdot n^{\log 3 - \log 2} - 2cn$

$\left[\sum_{i=0}^{n} r^i = \frac{1 - r^n}{1 - r}, \quad r \neq 1 \right.$

$\left[a^{\log b} = b^{\log a}, \quad a > 1, \ b > 1 \right.$

$= 2cn \cdot n \qquad - 2cn$

$= 2cn \cdot n^{\log 3 - \log 2} - 2cn$

$= 2c\cancel{n} \cdot n^{\log 3} \cdot \cancel{n}^{-1} - 2cn$

$= 2cn^{\log 3} - 2cn$

$= \Theta(n^{\log 3})$

4) *Inductive hypothesis:* $T(n) = 2cn^{\log 3} - 2cn$

$T(n) = 3T(n/2) + cn$ *using ind. hyp.*

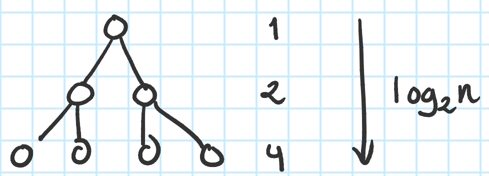
$= 3\left(2c(n/2)^{\log 3} - 2\frac{cn}{2}\right) + cn$

$= \frac{6}{2^{\log 3}} cn^{\log 3} - 3cn + cn$

$= \frac{6}{3^{\log 2}} cn^{\log 3} - 2cn$

$= 2cn^{\log 3} - 2cn$, *confirming the hyp.*

5)



$T(n) = 2T(n/2) + 1$

$$\sum_{j=0}^{\log n} 2^j = \frac{1 - 2^{\log n}}{1 - 2}$$

$= 2^{\log n} - 1$

$= n^{\log 2} - 1$

$= n - 1$