Questions from previous exams of TDT4120

1. Assume that you are given two integer arrays A = [a0, a1, … , an–1] and B = [b0, b1, … , bm–1]. You want to decide whether the array A can be partitioned into contiguous, non-empty, nonoverlapping segments such that each segment sums to one of the numbers in B. All numbers in A are to be part of exactly one segment. Several segments may sum to the same number in B and not all numbers in B need be used.

   Example: A = [2, 4, 5, 1, 2, 2, 9, –5], B = [6, 8]. The answer here is yes. (Possible partition: sum(2, 4) = 6, sum(5, 1, 2) = 8, sum(2, 9, –5) = 6.) Note: You are not required to find the partition itself. You only need to decide whether such a partition is possible or not. o)

   Describe an algorithm that solves the problem as efficiently as possible. What is the running time in the worst case? (Give your answer in asymptotic notation.)

2.
   a. What is the minimum and maximum number of edges in a connected, undirected graph with n nodes?
   b. What is the minimum and maximum number of nodes in a rooted binary tree of depth n? (The depth is the maximum number of edges in any path from the root.)

3. You have n friends. You know that some of your friends hate each other. Hatred is always mutual. You know exactly which pairs of friends that hate each other. You wish to unfriend some of your friends, so that none of your remaining friends hate each other. Your task is to determine if you can solve this problem by unfriending k friends, where k is an input parameter. You can assume that this decision problem is in NP. Show that it is NP-complete. Explain your reasoning clearly, and make sure you include all required parts of such an argument.

4. Multiplication of a matrix with n rows and m columns by a matrix with m rows and k columns has a running time of O(nmk). The algorithm for solving the 0/1 knapsack problem where one is to steal n objects using a knapsack with a capacity of k runs in O(nk). The first algorithm has a polynomial running time, while the second one has an exponential running time. How do you explain this?

5. Let A be the set of maximal acyclic undirected graphs and let B be the set of minimal connected acyclic undirected graphs, for arbitrary node sets, where “maximal” and “minimal” refer to the number of edges. What is the relationship between A and B? Explain briefly.

6. Your friend Lurvik has invented an algorithm. His algorithm can take a sequence of length n, where n is a multiple of k, and sort it into k equal-length segments, so that the elements in a given segment are not necessarily sorted with respect to each other, but all elements in any segment are greater than or equal those in all segments to the left, and less than or equal to those in segments to the right. Give a lower bound for the running time of the algorithm. Explain your reasoning briefly.

7. You are given a flow network with an associated flow (that is, a flow function—not just the flow value). Briefly describe an algorithm for determining whether the flow is maximum. What would the running time be?

8. (a) Which problem does Floyd-Warshall solve? What assumption must you or can you make, and how do they affect the running time? Explain briefly.

   (b) Solve the recurrence $T(n) = 2T(n/2) + T(n)/2 + n$, where $T(1) = 1$. Give your answer in asymptotic notation.

9. An algorithm processes an image in the form of an $n \times n$ matrix of pixels, by splitting it up into non-overlapping squares of size $(n/2) \times (n/2)$, processing these recursively, and then combining the results in linear time, as the function of the number of pixels involved. What is the total running time, as a function of n?

10. Consider the balanced partition problem, where we are given n integers $a1, \ldots, an$ in the range $0 \ldots k$, and we want to partition them into two sets S1 and S2 so that $\text{sum}_{x \in S1} x - \text{sum}_{y \in S2} y$ is minimized. (a) Show that the problem is NP-hard; (b) Sketch an algorithm for solving the problem in pseudopolynomial time.